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A METHOD FOR THE DETERMINATION OF OPTIMAL ORGANIZATION SIZE

FREDERICK H. KRUCK

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by

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Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE

United States Naval Postgraduate School
Monterey, California

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ABSTRACT

The rapid development of new weapons for the armed forces calls for changes in the tactics employed and in the organization sizes using these new weapons; in certain cases completely new organizations must be developed to employ the new weapon.

A method for the determination of optimal organization size is presented with particular emphasis on an organization employing a surface-to-air missile in support of ground combat troops. The method with appropriate modifications can be applied to any currently operational weapon and to new weapons in the development stage.

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TABLE OF SYMBOLS

(listed in order of appearance in text)

X	range of initial detection of a target
f	probability density function associated with X
n	total number of teams or weapons available
m	total number of sul divisions of a defended area
S_i	a subdivision of a defended area
t_i	expected number of targets on or over strip S_i
T	total number of targets expected per unit time
R_o	maximum range at which observation is possible
r_o	minimum range at which detection must be made
p_d	probability of detection in time to prepare and launch a missile at a target
p_k	probability of one missile destroying one target
$p_{s/x}$	conditional probability of successful launch and flight, given an initial detection range X
$p_{h/x}$	conditional probability that a missile hits and destroys a target, given a successful launch and flight and initial detection range X
f_i	degredation factor for strip S_i
P_n	probability that a target is destroyed if n missiles are fired at it
z	number of missiles fired at the aircraft designated L in a raid
$P(w)$	probability of destroying exactly w aircraft in a raid by a section

CHAPTER I

INTRODUCTION

The development of new weapons for the armed forces has been moving ahead at a very rapid rate since the end of the Korean War. The major emphasis has been in the development of missiles; we now have surface-to-surface, surface-to-air, air-to-air, and air-to-surface missiles. Considering both the surface-to-air and the surface-to-surface missiles, it is known that work has been done in the development of both types capable of being fired by small groups of infantrymen against attacking aircraft or tanks.

The adoption of a missile of the type discussed above creates immediate problems concerned with the employment of the missile and the organization which is to 'man' the missile. Some of these problems are stated below.

"How many missiles should be allocated to a Marine Division?"

"How many missiles should be given to each battalion and regiment of the division?"

"Should a separate organization be formed to employ the missile; if not, should additional personnel be assigned to an organization to man the weapon?"

"What kind of an organization should be developed?"

"How should such an organization be employed?"

The purpose of this paper is to develop a method which can be applied in the determination of the optimal organization size used to 'man' a weapon. Organization size as used in this sense will refer to squad size which is to be composed of an unknown number of teams. A team will consist of the personnel necessary to detect and acquire a

target, and prepare and launch a missile or salvo of missiles.

The problems of command and command control will not be discussed although methods will be alluded to when required to show the effect of various fire plans on the effectiveness of organization size.

A hypothetical situation will be considered and by a study of this situation a method of determining the optimal organization size will be developed. To simplify the discussion and to illustrate the method, attention will be focused on a surface-to-air missile; analogus arguments can be used to discuss surface-to-surface missiles or for that matter, any weapon which may be developed.

The hypothetical situation will consider an infantry unit which is in a defensive position and under air attack. Several different organizations are placed within the defensive perimeter of the unit, and for each of these, the distribution of attacking aircraft is studied and a determination of the effectiveness of the organization is made. To aid in realism, suitable ammunition restrictions are imposed.

To illustrate some of the problems encountered in reaching a final decision as to the best organization size, two criteria are used for the comparison of the effectiveness of the organizations studied.

The first criterion or 'measure of effectiveness' used is the percent of attacking aircraft which the organization expects to destroy. The second criterion consists of the probability function that describes the probability that the organization will destroy exactly none, one, or two of a pair of attacking aircraft since the threat to be met is considered to be comprised of several attacks of two aircraft each.

To illustrate the methodology, a specific example is introduced. The example is based on the use of three different organizations: six single

independently operating teams, three two-team squads, and two three-team squads. Two plans for the control of firing sequences are considered for the two-team squads, and just one plan for the three-team squads.

A total of twelve missiles are considered as the ammunition available for the defense of a unit against attack by a total of eight enemy aircraft. The results for this example are presented in tabular form in tables 1 and 2.

Independent Teams	Two Team Squads		Three Team Squads
	Fire Plan I	Fire Plan II	
37%	39%	53%	52%

Percent of attacking aircraft destroyed

Table 1.

Probability of destroying	Independent Teams	Two Team Squads		Three Team Squads
		Fire Plan I	Fire Plan II	
no aircraft	.048	.048	.245	.216
one aircraft	.432	.343	.470	.528
two aircraft	.520	.608	.285	.256
at least one aircraft	.952	.952	.755	.784

Probability of destroying aircraft in an attacking section

Table 2.

The method for determining the entries in the tables is discussed. In this particular situation the choice of the 'best' organization should be made after a careful study of the statement of the mission of the organization, the appropriateness and realism of the assumptions made, and the values in the tables. This final determination can involve utility theory and decision theory.

CHAPTER II

DEFINITIONS AND VARIABLES

1. Definitions

Definitions are an integral part of a problem. Many commonly used words, even technical words, mean different things to different people. Certain terms used in this paper are not in common usage and hence must be defined to avoid misinterpretation of the results.

The term 'team' will be used to refer to the personnel required to man a weapon. A team consists of those personnel required to detect and acquire a target, and prepare and launch a missile or salvo of missiles.

The term 'squad' will be used to refer to two or more teams each of which has control of a missile or group of missiles. As an example of this definition a machine gun section would be called a squad and a machine gun squad would be called a team.

'Section' as used herein and applied to aircraft will refer to two aircraft operating together and flying either side by side or one behind the other. Let L and R refer to the 'left' and 'right' or 'lead' and 'rear' aircraft, as appropriate.

'Group' will refer to two sections of aircraft, the second section following the first section by a very short time interval.

'Lateral range' is defined as the perpendicular distance from the weapon to the path of the target, provided that the target continues on a straight line course.

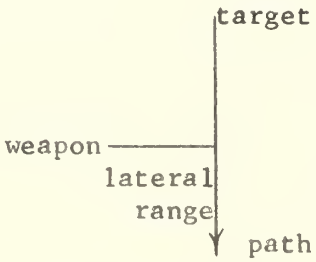


Diagram of lateral range

Figure 1.

2. Variables

The study of a problem can involve many factors each of which can take on several different values. They are called variables. To determine the optimal size organization for a new weapon it is necessary to consider each factor and determine its importance. Some of these can be neglected on the basis of apriori considerations whereas others must be introduced into the model being built.

The variables may be further classified as choice (or fixed) variables, chance (or random) variables, and nuisance variables. Choice variables are those which can be fixed or chosen by the investigator. Chance variables may not be specified precisely by the investigator; however, he may be able to specify the distribution of such a variable. Nuisance variables, as the name implies, are variables not of particular interest of themselves but whose influence must be admitted and accounted for.

The factors to be considered here will be introduced in general terms and then their application to the particular situation under study will be discussed. The following list is not all-inclusive, but contains the major factors considered to be important in this study.

A. Threat. The actual distribution or the expected distribution of targets that will be encountered can sometimes be deduced from the particular situation being studied. Paths of attack against friendly positions are usually restricted by terrain configurations and the intelligence available to the enemy. The particular types of distributions that will be used in this study are discussed in Chapter III. Some of the components of the distribution of threat are worthy of specific mention. Some are choice variables, others are random variables. They are

(1) Type of target. (A choice variable in this study).

The type of target can cause variations in kill probabilities, ranges of engagement, and tactics employed. In the case of tanks this could be the type of tank; light, medium, or heavy; the amount of armor plate, etc. In the particular case of an anti-aircraft missile, this would include a study of high performance jet aircraft, high performance conventional aircraft, and helicopters. In certain studies this could include a further breakdown according to the number of engines on the aircraft.

(2) Altitude of target in relation to the weapon. (This is fixed at the value least favorable to the defense in this study). This is a variable that has required consideration by all conventional long range weapons such as mortars, howitzers, and large caliber guns. In all of these weapons if corrections are not made for the difference in altitude between the weapon and the target, the initial rounds will not be 'on target'. The cost of missiles being far greater than the cost of conventional high explosive shells and the supply of missiles generally more limited, it is necessary to consider all possible factors before launching a missile.

The primary effect of altitude of aircraft in relation to the weapon is the influence on the range at which observation (detection) is possible, which in turn is a determining factor in the probability of detection.

(3) Velocity of target. (This is fixed at the value least favorable to the defense in this study). It would be reasonable to assume that the enemy will travel at a speed which is optimal for his mission. This is a factor determining the amount of time that a target will be within range of the weapon.

For the case of vehicles it is known that they will travel at speeds consistent with conditions; i.e., fast if under known

observation or exposed to enemy fire, slower if not observable by the enemy but where high speed could raise dust which would be visible to the enemy.

Aircraft have a speed which is optimal for the type of attack being carried out; this is not necessarily the same for all types of attack.

(4) Number of targets. (This is fixed as a daily or per attack rate in this study). The number of targets in a single attack or the number of targets expected to be encountered in a single day is an important factor in determining the effectiveness of a particular organization. A method of dovetailing organization size, number of targets, and the availability of ammunition must be considered.

(5) Lateral range to the target. (A random variable). Every weapon has a specific maximum (effective) range. This can be considered as a circle of proper radius drawn around the location of the weapon. If the target traverses a diameter of the circle, then the target will be in range the maximum amount of time; the lateral range in this case is zero. As the lateral range increases, the target travels along a chord of the range circle; hence there will be less time available to intercept and destroy the target. The degradation of kill probabilities as a result of increasing lateral range is a most important factor in a study of this type.

B. Range to the target at initial detection. (A random variable). Let this random variable be X and its associated density function be f. This is the primary factor which determines the probability of detection of a target such that there is sufficient time for a missile to be prepared and launched.

C. Ammunition available. (This is fixed as a daily or per attack rate in this study). Limitations placed on the amount of ammunition available can cause changes in the effectiveness of an organization. Variations in the daily ammunition supply rate can become important in the overall logistics picture for a new weapon. In a study concerning logistics this supply rate might best be studied as a random variable.

D. Organization size. (A choice variable.) This is the item being studied, hence it is the primary variable. It must be varied and the results of these variations carefully compared.

E. Nuisance variables. There is a certain class of variables which enter into any study of tactics or organization size and these variables are difficult, if not impossible, to evaluate. No real yardstick has been derived to evaluate them, yet they appear in problems of this type and can have an important influence on the conclusions reached. This is particularly true if an experiment is designed to compare the theoretical results of a study such as this with actual field trials. Some of these variables are:

(1) Terrain

(2) Morale of the participating troops

(3) State of training of the participating troops.

Theoretical studies cannot evaluate these variables, they are mentioned merely because they must somehow be included in any final decision reached as the result of experimentation based on a study of this type.

CHAPTER III

ASSUMPTIONS

1. General

A study which is being undertaken may at first appear to admit to no method of solution because of the complexity introduced by the many variables. Assumptions are therefore introduced and serve to deliniate and limit a problem being studied by reducing the complexities in the variables; in addition, they can serve to reduce the magnitude of the computations required. The validity of the conclusions is however a function of the reasonableness of the assumptions made in the study. The assumptions to be made in this study will be used to reduce the number of variables that must be considered by eliminating foolish tactics and illogical actions on the part of the opposing forces.

The object of this study, as stated in the introduction is to obtain a method to determine the optimal size of an organization which is to man a weapon being developed. The general method will focus attention on a surface-to-air missile. The general method will be illustrated in a later chapter when assumptions about a specific hypothetical situation will be set up and a solution obtained.

The assumptions to be made in this study can be grouped into several logical categories as follows:

A. Assumptions about the organization

B. Assumptions about the threat

C. Miscellaneous assumptions.

2. Assumptions about the organization

Military organizations being compared cannot be properly evaluated without placement in some kind of a tactical situation such as that in

which the organization would be called upon to perform its mission. The general situation in which the organizations being compared would actually be performing their mission can be specified by the use of assumptions. Limitations on the ability of the organizations to perform certain assignments can be specified, and the necessary time space factors set up with the use of assumptions.

A. Organizational assumption number one. An infantry unit can be engaged in several tactical missions, the major ones being offensive combat and defensive combat.

The unit in an attack situation will normally be assigned a frontage commensurate with the mission assigned. A situation such as this involves combat of a very fluid type. It is also hoped that local air superiority will be maintained by friendly forces so that the possibility of air attacks by enemy aircraft is extremely unlikely. For these reasons, the relation of organization size to offensive combat will not be studied; only situations involving the unit in the defense will be studied.

B. Organizational assumption number two. Non-nuclear warfare places neighboring units in close proximity to each other thus resulting in mutual support of units and integrated air defense plans. As a result of this, aircraft attacking a specific unit can be taken under fire by weapons assigned to neighboring units. The resulting increase in overall air defense effectiveness in this situation would produce an apparent increase in effectiveness for the organization which would be lost if the unit were isolated from its neighbors. Hence, it seems desirable to restrict attention to providing a good air defense under the least favorable conditions.

Assume therefore that maximum dispersion will be employed in any tactical situation. In this case there is a definite possibility that there will be undefended corridors between adjacent units. With sufficient intelligence the enemy will utilize these corridors to penetrate any air defense to attack the unit. Since the actual air defense of the unit will be isolated from that of neighboring units, the effectiveness of the new air defense organization can be measured without being influenced by other similar organizations.

C. Organizational Assumption number three. Some assumptions must be made concerning the size and nature of the organization which will employ the new weapon. Size can be described by the total number of teams, n. Organization is the basic variable being examined. For any given value of n, several different organizations can be assumed and then compared.

D. Organizational Assumption number four. Assume that for any raid, regardless of size, every team fires one missile or one salvo of missiles. If this assumption is not made, then there is the problem resulting from the fact that the first team to take a target under fire may destroy the target. This problem can complicate a study because there is a resultant saving in the number of missiles that may be expended against a single raid and the effectiveness of the organizations being compared can be materially altered. Gaming can be an effective tool if this assumption is not made and effectiveness of organizations is to be studied.

E. Organizational assumption number five. A realistic restriction must be made on the amount of ammunition available per weapon per day. This could be a specific number of rounds or a specific number of salvos. Assume that a single firing by a weapon is a unit and that no distinction is made between a single missile and a salvo of missiles. Mixed salvos will not be considered.

3. Assumptions about the threat.

A tactical situation having been assumed for the organizations to be compared, it is necessary to present the organization with some sort of a threat against which it must take action. The enemy does not have unlimited resources. The production effort of the enemy can limit the threat facing the organization. Even if the factors of resources and production effort are not considered, some assumptions about the threat are necessary so that each different size organization being considered will be forced to react to the same identical threat and thus a fair comparison of organizations can be made.

A. Threat assumption number one. An integrated air defense plan must provide for defense against all types of air attack; this includes high performance jet aircraft, high performance conventional aircraft, helicopters, and vertical takeoff aircraft.

It will be assumed that the only threat by aircraft will be by either propellor driven or jet aircraft. Vertical takeoff aircraft are at present still in an experimental and development stage. Helicopters should be considered in a separate study because of their unique flight characteristics.

It is the opinion of the author that helicopters would require a different approach for a study of this type.

B. Threat assumption number two. The probability of detection of aircraft is a function of several variables; one of these is the altitude of the aircraft relative to the surrounding terrain and relative to the weapon. Aircraft flying just above tree top level are more difficult to detect than those at higher altitudes where the effects of contrasting background can increase the chances of detection. It would be possible to

choose several different altitudes for the aircraft and compute the effectiveness of each different size organization for each altitude chosen; however, it is wise to assume that the enemy will adopt his best tactics and fly at an altitude which will be consistent with his mission and which will also reduce the probability of detection to a minimum.

Therefore it will suffice to assume a single altitude for all attacking aircraft in a given situation.

If such an assumption is not made, then the distribution of the actual altitude of the attacking aircraft must be investigated and this distribution included in the model.

C. Threat Assumption number three. It has already been noted that the random variable that describes the range of initial detection of aircraft by a team, X, plays an important role in this study. The distribution of X can be assumed, empirically estimated, or derived from physical considerations.

It is here assumed that X is a continuous random variable with a density function f. It is further assumed that f is the same for all weapons locations within the area defended by the organization being examined.

D. Threat Assumption number four. To study the effectiveness of various size organizations it is necessary to pose a threat of some kind so that comparisons of the organizations can be made. This can be a daily threat or a threat for a specific time period. In the case of aircraft this can be the number of aircraft expected to attack the area in unit time, the number of attacking sections per unit time, or the total number of raids per unit time.

E. Threat Assumption number five. It is possible to consider attacks by single aircraft; however, aircraft usually attack in sections, one aircraft affording protection to the other. Based on this it will be reasonable to assume that aircraft attack only in sections and groups as defined in the introduction.

F. Threat Assumption number six. Since all targets will not appear at the same location relative to a weapon, a distribution of incoming targets must be assumed. The distribution may be obtained from a consideration of the tactical situation.

In the case of attacking aircraft, it will be further assumed that the aircraft fly in straight lines when actually conducting an attack, and that the flight path will be parallel to the front lines.

4. Miscellaneous assumption

The nuisance variables can be discounted in a theoretical study of this type by assuming identical values for these corresponding to each different set of factors which will actually be varied.

CHAPTER IV

GENERAL METHODOLOGY

The method to be presented here is completely general and is applicable to weapons of many types. It may be used for studies of weapons organizations aboard ships provided that the proper minor modifications are made in some of the variables. Evaluation of criteria may be obtained for a particular case by using the values for the weapon under consideration and the desired values for the choice variables. This will be illustrated with a hypothetical example in Chapter V.

This paper will concentrate on a surface-to-air missile.

As a result, targets will be considered as attacking aircraft; note, however, that only very minor modifications in some of the words used would make the paper applicable to other type targets.

First, threat is examined.

The defended area under consideration can be subdivided into strips parallel to the front lines. Let m be the total number of such strips and let the strips be designed as S_i where $i = 1,2,3.....m$. Let S_1 be the strip closest to the front lines and S_m the strip farthest from the front lines.

The distribution of attackers can now be approximated by a step function over the strips where the height of the function over the ith strip is proportional to the probability that a given attacker will attack along strip S_i. The number of targets expected to be observed over strip S_i or expected to attack along strip S_i in a given raid or on a given day can therefore be designated as t_i where $i = 1,2,...m$ and

$$\sum_{i=1}^{m} t_i = T = \text{total number of targets expected to attack the area in 'some' time interval.}$$

It is convenient to express the expected number of targets for each strip in tabular form.

Strip	Expected number of targets t_i
S_1	t_1
.	.
.	.
.	.
.	.
S_m	t_m

Comparison of strips and targets

Figure 2.

Next, attention is turned to the range of initial detection.

Terrain will determine the maximum range at which observation of the attackers will first be possible. It has previously been assumed that this range is the same for all weapon locations; note however, that in an actual situation the weapons would be placed so that they may best accomplish their mission. In such a case the detection range could be different for each weapon location and the probability distribution of the range of initial detection would be computed separately for each weapon location.

Detection at the maximum range at which observation is possible will occur only under the most ideal conditions; however, there is an increase in the probability of detection as the target approaches the weapon position.

Let R_o be the maximum range at which observation is possible. The probability that initial detection by a team will take place at range r, or greater, may be expressed as

$$\int_r^{R_o} f(x)\ dx$$

where the function f is the density function introduced in Threat Assumption number three.

Let r_o be the minimum range at which detection must be made if there is to be sufficient time to prepare the missile or weapon for firing, launch the missile, and allow it to intercept the target.

Let p_d be the probability of detection in time, i.e., prior to r_o, by one team. Then

$$p_d = \int_{r_o}^{R_o} f(x) \quad dx$$

Weapon behavior is usually incorporated with the range of initial detection to yield the fundamental quantity p_k: the probability that one missile or one salvo of missiles destroys one target. In addition to the range of detection, p_k must reflect the mechanical behavior of the missile at the launch position and during its flight to the target; further, p_k is influenced by guidance characteristics that may not inevitably lead the missile to intercept the target; finally, the lethality of the warhead must be considered. Each of these additional factors influencing p_k is clearly dependent on the flight path of the missile. In cases where the target follows a path directly at or over the weapon site, the dependence can be expressed in terms of the range from the weapon to the target. If the time from detection to launch is also known, the dependence can be expressed in terms of the range of initial detection, X. In such cases, the determination of p_k can best be accomplished by combining the appropriate conditional probabilities. Formally this can be written as shown below:

Let $p_{s|x}$ be the probability of a successful launch and a successful flight to the target, given that initial detection occured at range x.

Let $p_{h/x}$ be the probability that the missile hits and destroys the target, given a successful launch and flight and initial range of detection, x.

Then, p_k can be obtained by computing

$$p_k = \int_{r_o}^{R_o} p_{s/x} \; p_{h/x} \; f(x) \; dx .$$

If the target does not follow a path directly at or over the weapon site p_k may be adjusted to account for the effects of lateral range, if required. There is a relation between range to a target and lateral range such that the probabilities which are functions of range may be effected by the lateral range. In the case of an actual missile, any changes in p_k can be determined by careful analysis of $p_{s/x}$, $p_{h/x}$, and p_d. The value of p_k may increase, decrease, or remain the same. The change in p_k may be a simple linear function or a more complicated function.

In many cases, a factor called a degradation factor, f_i say, can be computed for application to the values of p_k to be applied to the targets along strip S_i. The factor f_i will be greater than one if it increases the probability of a kill, less than one if it decreases the probability of a kill, and equal to one if it leaves p_k unaltered. The probability of destroying a target in strip S_i then can be written as

$$p_k \; f_i \; .$$

Clearly if the number of rounds fired at a target is increased without limit, the probability of kill will approach one.

The degradation factor will influence weapon placement in a defended area and it appears to be intuitively appealing to locate the weapons in the strip where the greatest number of targets are expected to attack.

The problem of obtaining some actual values for the probability of kill, p_k, and the degradation factors, f_i, must now be considered. The ideal solution to this problem is an experiment conducted in a realistic setting. If the experiment is designed so that in addition to it being realistic in a tactical sense it is also carefully controlled so that actual data as to the ranges and times is obtained, then the values of p_k and f_i can be determined as a result of the experiment. It is realized that cost can be a determining factor in the conduct of an experiment and several 'runs' must usually be made if good data is to be obtained for the determination of p_k and f_i. If an experiment cannot be conducted, p_k might be obtained from values of $p_{s/x}$ and $p_{h/x}$ which might be estimated from the engineering requirements for the missile. Pre-production tests usually made by the manufacturer can also provide a basis for the determination of $p_{s/x}$ and $p_{h/x}$. Note that tactical realism may be lost if an experiment is not conducted.

Clearly the probability of destroying a target can be increased if the number of missiles fired at the target is increased. Consider the relation between the number of missiles fired at a target and the kill probabilities. Assuming that the firings by the weapons are independent, the relationship can be developed as follows:

Let $1 - p_k$ be the probability that one missile or salvo of missiles does not destroy the target.

Now $(1 - p_k)^n$ is the probability that n missiles do not destroy a target.

Then $1 - (1 - p_k)^n$ is the probability that a target is destroyed if n missiles are fired at it.

Let $1 - (1 - p_k)^n$ be designated by P_n.

The basic calculations have now been completed and the different organization sizes can now be investigated. Since attacks are organized in sections or groups (Threat Assumption number five), the analysis will also be carried out with respect to sections and groups of attacking aircraft.

For each different size organization being considered, certain computations must now be made; in addition, for the organizations consisting of more than single teams it will be necessary to propose several fire plans so that the best missile allocation against all targets is assured. In the case of a surface-to-air missile, analysis of the fire plan that is proposed for an organization which does not consist of individual independently operating teams will determine how many missiles will be launched at each aircraft in an attack by a section or group of aircraft.

To describe a fire plan, the aircraft in a section are designated as L and R corresponding to their location in relation to the weapons.

If the n weapons operate independently, then the missile allocations will be left to chance. The probability that a specific number of missiles will be launched at one of the aircraft can be computed. Consider the case where z missiles are launched at the aircraft designated as L; then n - z missiles will be launched at the aircraft designated as R, and the missile allocation in such a case will be designated as z, n - z. There are n + 1 missile allocations possible. The probabilities of the shot allocations for the case of independent action by the teams can be computed using the binomial distribution. In this case of independent action it is reasonable to assume that p = q = 1/2 so that

$$\binom{n}{z} p^z q^{n-z} \quad \text{is} \quad \binom{n}{z} \left(\frac{1}{2}\right)^n.$$

This probability can be computed for every missile allocation possible in the case of independent team action.

The expected number of aircraft destroyed in an attacking section for any missile allocation can be computed. Let this be called the 'pay-off' for a particular shot allocation. Symbolically the payoff is equal to

$$0(1-P_z)\ (1-P_{n-z}) + 1(1-P_z)P_{n-z} + 1(1-P_{n-z})P_z + 2P_z\ P_{n-z}$$

and this can be reduced to

$$P_z + P_{n-z} \qquad \text{where } z = 1,2,3,\ldots\ldots n.$$

The last two computations can be presented in tabular form as shown in figure 4.

Allocation of missiles	Probability of Allocation	Payoff
0, n	$\binom{n}{0}\left(\frac{1}{2}\right)^n$	$P_o + P_n$
1, n - 1	$\binom{n}{1}\left(\frac{1}{2}\right)^n$	$P_1 + P_{n-1}$
. .		. .
. .		. .
. .		. .
r, n - r	$\binom{n}{r}\left(\frac{1}{2}\right)^n$	$P_r + P_{n-r}$
. .		. .
. .		. .
. .		. .
n, 0	$\binom{n}{n}\left(\frac{1}{2}\right)^n$	$P_n + P_o$

Allocation probabilities and Payoffs

Figure 3.

To compare the organizations, a measure of effectiveness should be defined. The use of a measure of effectiveness allows for objective numerical comparison of the organizations as the factors are varied.

Generally several different measures of effectiveness could be conceived so that the problem of finding the best measure is ever present. Analysis of the mission of the organization being studied will generally lead to a preference for one or several of the measures of effectiveness.

As noted in the introduction, two measures of effectiveness suggest themselves for the study of a surface-to-air missile. The first of these is the ratio of the total expected number of attackers destroyed to the total threat per unit time. That is,

$$\frac{E\ (\text{total targets destroyed})}{T}.$$

The second measure is the probability function for the number of aircraft destroyed per section of attacking aircraft, P(w).

In the case of independent team action,

$$P(w) = \sum_{i=0}^{n} \Pr(\text{shot allocation } i)\ P(w \text{ destroyed} \mid \text{shot allocation } i)$$

If it is now desired to use the first criteria for comparison of different size organizations, the expected number of aircraft destroyed for an attacking section can be computed from the relation

$$E(\text{aircraft killed}) = \sum \Pr(\text{shot allocation}) \times \text{Payoff for the allocation.}$$

In general, the degradation factor f_i is less than one and serves to reduce the payoff for some or all shot allocations. If it is neglected, the expectation computed above will then be the maximum value that can be achieved for such cases.

In certain cases the location of the weapons must be considered and the expectations computed separately for each weapon location. These cases will occur if:

(1) f_i is not sufficiently close to one or,

(2) f_i is not equal to one and few missiles are to be fired at each target.

In these cases the possibility of locating the weapons in each strip and in all combinations of strips must be considered. This will involve considerable additional computation.

The above computations must be repeated for successive attacking sections until either the expectations have been computed for the total threat or the ammunition allotment for the time period has been expended, whichever is sooner.

The effectiveness is then

$$\frac{\text{Sum for all attacking sections of the expected targets destroyed}}{T} .$$

If the second measure of effectiveness is to be used for the comparison of the organizations, the probability of destroying none, one, or two aircraft per attacking section must be calculated. This can be done using the formula on page 23, and would be computed instead of the expected number of aircraft destroyed.

The computational procedures discussed herein would be applied to each different organization being considered and the results presented in tabular form for analysis and possible decision as to the 'best' organization size. This final decision can involve utility theory and/or decision theory.

It should be remembered that any decision made at this point of the study will reflect the assumptions made in the study. The results thus far can be more carefully analyzed before a decision is reached if certain of the assumptions are either relaxed or completely eliminated. Clearly in this study the removal of ammunition restrictions, changes in the assumptions about the method of attack to be used by the enemy, and removal of the isolation of the organization assumption will result in changes of the

effectiveness of the organizations.

Experiments might be conducted to determine if the assumptions made are indeed realistic; in addition, an experiment could provide realistic values for some of the choice variables.

Analysis of the assumptions is now a logical next step before any final decision is reached.

CHAPTER V

AN EXAMPLE

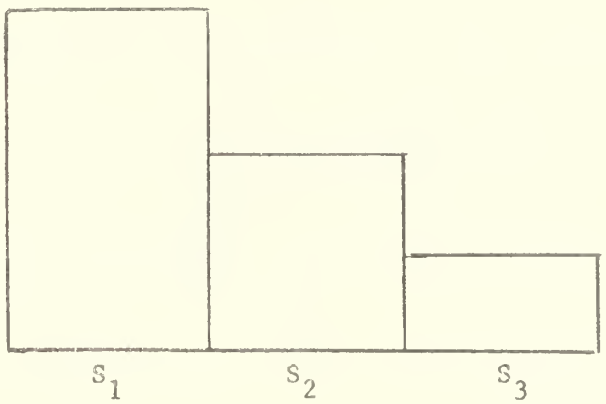
1. The Situation

The computations involved in the general method can now be illustrated with a specific hypothetical example. The example will illustrate the use of fire plans and their interaction with organization size. Some specific values for the choice variables will be required in this example.

The hypothetical example will consider an availability of six weapons ($n = 6$), a threat of eight aircraft per day ($T = 8$), and an ammunition availability of twelve missiles per day.

Let the defended area be divided into three strips of equal width. The strips will be designated S_1, S_2, and S_3.

Assume that the distribution of attacking aircraft is as shown in figure 4. This is an approximation to a triangular distribution such as might be encountered if the personnel in the defended area are actively engaged by enemy troops. Exchanges of fire would indicate the front lines to the enemy and hence most of the attacking aircraft would fly over strip S_1.



Distribution of attackers

Figure 4.

The expected number of targets in each strip can be computed. The results are shown in table 3.

Strip	Targets (t_i)
S_1	4 4/9
S_2	2 2/3
S_3	8/9

Threat per strip

Table 3.

To illustrate the computation of p_k, let the density function associated with the range of detection be $f(x)=e^{-x}$. In addition, let engineering requirements be such that $p_{s/x}$ and $p_{h/x}$ are independent of range of initial detection and are equal to .95 and .60 respectively. Let the range at which detection is first possible, R_o, be approximately two miles, i.e. 3400 yards; let the range at which detection must be made if there is to be sufficient time to launch a missile be 300 yards. The probability of kill, p_k, is then computed as follows:

$$p_k = \int_{R_o}^{} p_{s/x} \; p_{h/x} \; f(x) \; dx$$

$$= \int_{.3}^{3.4} (.95)(.60) \; e^{-x} \; dx$$

$$= .40$$

The degradation factor will be assumed negligible, i.e. $f_i = 1$ for all i.

Using the relation

$$P_n = 1 - (1 - p_k)^n$$

and letting n = 1,2,...6, the following probabilities of kill are computed.

n	P_n
1	.40
2	.64
3	.78
4	.87
5	.92
6	.95

Values of P_n for $p_k = .40$

Table 4.

Three organization sizes will be considered: independent teams, two team squads, and three team squads. Hypothetical fire plans will be introduced as appropriate.

2. First Criterion

The first measure of effectiveness of the organizations computed for this example will be the expected percentage of aircraft destroyed per day. This is the first criterion of the general method.

A. Independent teams. This organization expends six missiles against each attacking section so that only two sections of aircraft can be taken under fire. The expected number of aircraft destroyed in the second attacking section will be the same as for the first section.

Note that in attacks by groups of aircraft the results will not change since a maximum of four aircraft will be taken under fire.

Independent action by the six teams which are all placed in S_1 allows for seven different missile allocations to an attacking section. If the teams do act independently, then each is free to choose as its target either the aircraft designated as L or the aircraft designated as R, and one choice is as likely as the other. If the binomial distribution is now used to compute the probability of missile allocations, $p = (1 - p) = 1/2$. The computation is illustrated for $\not{z} = 4$ and hence $n - \not{z} = 2$.

$$\binom{n}{\not{z}} \; p^{\not{z}} \quad (1 - p)^{n-\not{z}} \quad \text{becomes}$$

$$\frac{6!}{4!\,2!}\left(\frac{1}{2}\right)^6$$

$$\frac{15}{64}$$

The payoff in the 4,2 allocation is

$$P_4 + P_2 = .87 + .64$$

$$= 1.51$$

The results of the above computations are shown in table 5.

Allocation	Probability	Payoff
0,6	1/64	.95
1,5	6/64	1.32
2,4	15/64	1.51
3,3	20/64	1.56
4,2	15/64	1.51
5,1	6/64	1.32
6,0	1/64	.95

Allocation Probabilities and Payoffs

Table 5.

The expected number of aircraft destroyed can now be computed for an attacking section of aircraft. This expectation is 1.47 aircraft.

The effectiveness of this organization is

$$\frac{2.94}{8} = .37$$

B. Two Team Squads. The six teams are now formed into three squads of two teams each. Fire plans can now be introduced since the teams in a squad can act together to improve the overall effectiveness of the organization.

(1) Fire Plan I. Let the teams of each squad be designated as team #1 and team #2. Let the attack policy be 'all teams #1 will attack the right aircraft and all teams #2 attack the left aircraft'. This will ensure that 3 missiles are launched at each aircraft. Using the ammunition restriction imposed for this example, the disadvantage that only four aircraft can be taken under fire is readily apparent.

The missile allocation against a section will be 3,3 and the expected number of aircraft destroyed will be 1.56.

The second attacking section will receive the same missile allocation and the expected number of aircraft destroyed will be the same.

The maximum expected number of aircraft that are destroyed if the enemy attacks in sections will be 3.12 aircraft and this yields an effectiveness for this organization and fire plan of .39.

In the case of attacks by groups of aircraft, the results will be the same as for attacks by sections since only the first section in each group will be taken under fire.

(2) Fire Plan II. Let this be, "Squads one and two are to take the first section of aircraft under fire, squad three is to take the second section under fire, squad one the third section, and squads two and three the fourth section. Fire plan I is to remain in effect to determine missile allocations". This fire plan will ensure that all eight aircraft are taken under fire independent of the method of attack used by the enemy.

To determine the effectiveness of this organization and this fire plan, each attacking section must be considered separately.

The first section attacking the defensive area will be taken under fire by squads one and two with two rounds being expended against each aircraft. The maximum expectation in aircraft destroyed will be 1.32.

The second attacking section will be taken under fire by squad three and only one round will be expended against each aircraft. The maximum expectation in aircraft destroyed will be .80.

Note that in the case of attack by a group of aircraft both sections will be taken under attack exactly as if the attack were made by two separate sections. The maximum total expectation for the first group or the first two sections will be 2.12 aircraft.

The third section will be taken under fire by squad one with one missile launched at each aircraft and a maximum expected .80 aircraft destroyed.

The fourth section will be taken under fire by squads two and three with two missiles launched at each aircraft and a maximum expected 1.32 aircraft destroyed.

The total number of aircraft expected to be destroyed

for attack by all eight aircraft is seen to be 4.24 for a maximum effectiveness of .53.

Clearly for an organization consisting of two team squads a maximum effectiveness can be obtained by the use of fire plan II.

C. Three Team Squads. The original six teams can now be formed into two squads of three teams each; only one simple fire plan will be used. 'Let the first squad take the first and third attacking sections under fire; the second squad, the second and fourth sections. To ensure that the best missile allocation is used let the first team in each squad take the left aircraft under fire, the second team the right aircraft, and let the third team choose either aircraft'.

The fire plan used by this organization ensures a missile allocation of 2,1 against each section and will result in the same effectiveness regardless of the method of attack used by the enemy. The expectation for each section will be 1.04 aircraft destroyed so that the maximum number of aircraft expected to be destroyed in any one day will be 4.16 for a maximum effectiveness of .52.

The results for the three organization in this example can be summarized in tabular form as shown in table 6.

Independent Teams	Two Team Squads		Three Team Squads
	Fire Plan I	Fire Plan II	
37%	39%	53%	52%

Effectiveness of Organization

Table 6.

If a determination must be made as to the best organization the choice would be either two team squads and fire Plan II, or three team squads. The problem in this case would be the difficult one of determining the worth of a one percent difference in the effectiveness of organizations.

3. Second Criterion

The mission of the organizations studied in this example can be stated as 'to provide an air defense capability to the defended area'. This mission must be kept in mind as conclusions are reached. Clearly, an air defense capability can be provided without actually destroying the attacking aircraft. If the enemy air attacks are materially reduced, then the organization could be said to be accomplishing its mission. The reduction in the number of attacks could be measured by the 'danger' presented to aircraft attacking the defended area. This 'danger' presented can be measured by the probability of destroying exactly none, one, or two aircraft.

This utilizes the second measure of effectiveness of the general methodology. These probabilities can be computed utilizing the relations

$$\Pr(0 \text{ aircraft destroyed}) = \sum \Pr(\text{shot allocation}) \times \Pr(\text{neither killed} \mid \text{allocation})$$

$$\Pr(1 \text{ aircraft destroyed}) = \sum \Pr(\text{shot allocation}) \times \Pr(\text{one killed} \mid \text{allocation})$$

$$\Pr(2 \text{ aircraft destroyed}) = \sum \Pr(\text{shot allocation}) \times \Pr(\text{both killed} \mid \text{allocation})$$

The results of this computation considering both organization size and fire plans in this example are summarized in table 7.

Prob. of Destroying	Independent Teams	Two Team Squads		Three Team Squads
		Fire		
		Plan I	Plan II	
No Aircraft	.048	.048	.245	.216
One Aircraft	.432	.343	.470	.528
Two Aircraft	.520	.608	.285	.256
At least one Aircraft	.952	.952	.755	.784

Summary of Probabilities

Table 7.

Careful analysis of the assumptions made in the study would now be required. Some of the assumptions could be varied or eliminated and the best possible organization size adopted after all eventualities are studied.

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